Using an orbital method and GPS measures of the ground control points in the georeference of the space images

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1. Introduction

In works [1,2], the rectification and the precise georeference of space images are examined, analyzed and mathematically grounded. For this purpose, many additional settings and requirements are made for the quantities used in the mathematical model, namely: the necessary spatial orthogonal coordinate systems are defined; the coordinates of the ground control points (GCP) P_j (j = 1,2,...,8) are determined by GPS measures; the Earth's /referent/ ellipsoid being assumed as projection plane is taken, with reading of the ellipsoid's heights (fig.1) [2] .In the present work, these settings will be accounted for , but the attention will be focused on the issues, related with the possibility of approximation and extrapolation of the orbital elements at the moment the image was taken t_k .

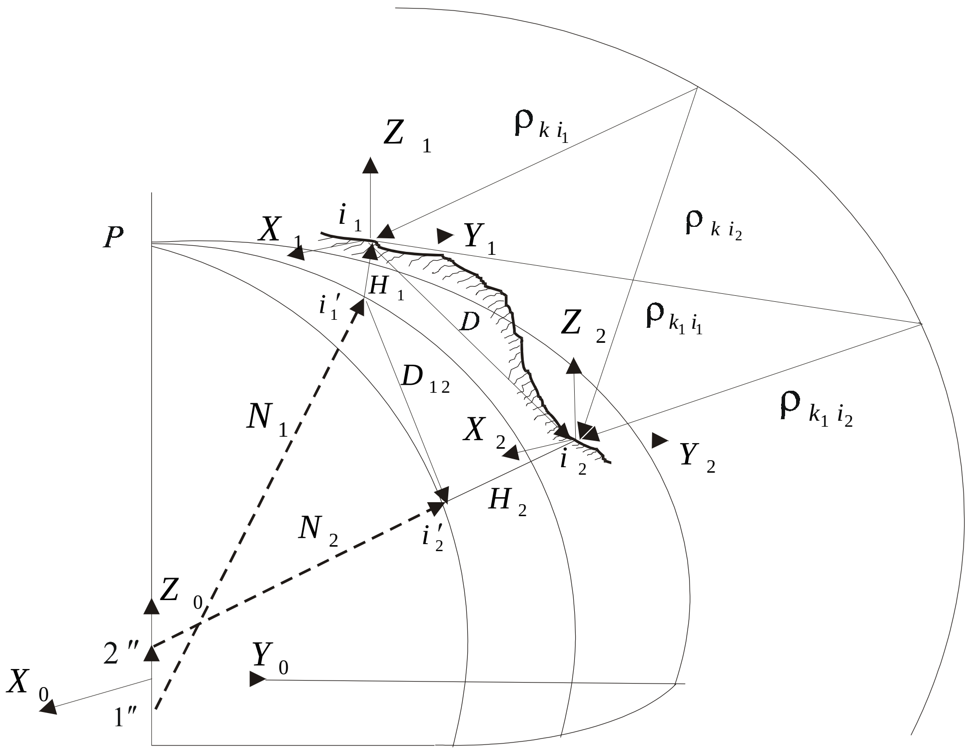

Fig.1

Each satellite is characterized in the space area by it's initial elements for a given starting epoch t_0 . These elements could be extrapolated and improved for the moment the image was taken t_k (k = 1, 2…n) with some of the experimental orbital or numerical methods, by using: Kepler's elements in the inertial coordinate system $\boldsymbol{F} = (i, \Omega, \omega, a, e, M)_0^T$; orthogonal inertial coordinates and components of velocity $\boldsymbol{R} = (x, y, z, \dot{x}, \dot{y}, \dot{z})_0^T$; spherical coordinates and components of velocities $\boldsymbol{S} = (\xi, \eta, \omega, \xi', \eta', \omega')_0^T$; or numerical integration of the fluxional equations of motion, and others [3, 4, 5 ,6].

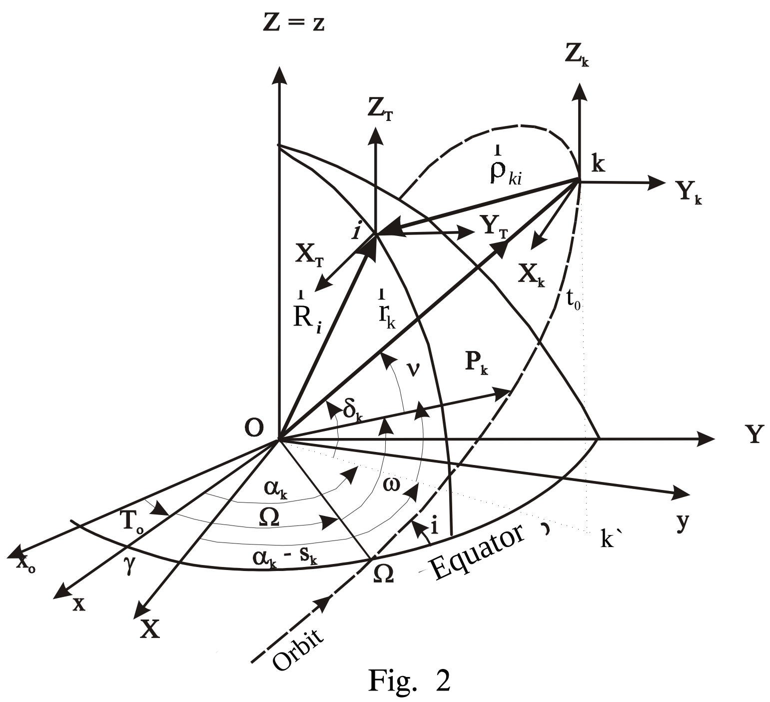

Fig. 2

We shall consider that in the first stage, by a chosen orbital method, an extrapolation of the initial elements of the satellite is made, and the initial quantities of the satellite at the moment of taking the picture t_k (k = 1,2…n) are determined. With these 'transferred' quantities and the of the known GCP determined by GPS coordinates, according to [2] fluxional specification of the coordinates of the satellite at the moment t_k , and the Oilers elements (Ω,i,ω,) could be made.

With the quantities thus obtained way, the orthogonal coordinates of the necessary control points of the image (fig.2) in Greenwich coordinate system could be determined, i.e. the already known GCP of the area, by which the geometrical deformations could be eliminated.

2. Approximation and extrapolation of the satellite's orbit, for the moment of taking the image t_k .

To solve the problem, the inter orbit of the satellites for short intervals of time in spherical coordinate system [4, 7, 8] will be used. This way of extrapolation of the initial elements, provides a precise way of determination the spherical coordinates of the satellite $\boldsymbol{S} = (\xi,\eta,\omega,\xi',\eta',\omega')_0^T$ which was used in the INTERCOSMOS program while a specific software for processing was also made.

The relation between the inter spherical coordinates $\boldsymbol{S_k} = (\xi,\eta,\omega)_k^T$ and the rectangle coordinates $\boldsymbol{r}_k = (X,Y,Z)_k^T$ is given by [3, 4, 7]:

$$(1)\qquad \begin{cases} X_k = \sqrt{(c^2+\xi_k{}^2)(1-\eta_k{}^2)}\ \cos\omega_k\ , \\ Y_k = \sqrt{(c^2+\xi_k{}^2)(1-\eta_k{}^2)}\ \sin\omega_k\ , \\ Z_k = c\sigma + \xi_k\eta_k\ . \end{cases}$$

To extrapolate of the orbit for the moment t_k, the expressions, providing to determine both the coordinates $(\xi,\eta,\omega)_k$ and the components of the velocities $(\xi',\eta',\omega')_k$ [4, 8] are used:

$$(2)\qquad \begin{cases} \xi_k = \sum_{i=0}^{n} a_{0i}\tau_k^i\ , & \xi_k{}' = \sum_{i=0}^{n} ia_{0i}\tau_k^{i-1} \\ \eta_k = \sum_{i=0}^{n} b_{0i}\tau_k^i\ , & \eta_k{}' = \sum_{i=0}^{n} ib_{0i}\tau_k^{i-1} \\ \omega_k = \sum_{i=0}^{n} c_{0i}\tau_k^i\ , & \omega_k{}' = \sum_{i=0}^{n} ic_{0i}\tau_k^{i-1}\ ; \end{cases}$$

$$(3)\qquad \tau = \sum_{i=1}^{n} I_{oi}\Delta t_k{}^i\ , \qquad \Delta t_k = t_k - t_0\ .$$

τ - is determined time; $I_i \cong \xi_0^2 + c^2\eta_0^2 + \varepsilon_0$.

The coordinates obtained for the moment of taking the image t_k , must be corrected, accounting for the gravitational and non-gravitational disturbances [4, 10,11]. By the used method , even the smallest disturbances, influencing the motion of the satellite could be determined, after which they are summed to the expressions:

(4) $$\delta\xi = \sum_{i=1}^{n}\delta\xi_i\ ; \quad \delta\eta = \sum_{i=1}^{n}\delta\eta_i; \quad \delta\omega = \sum_{i=1}^{n}\delta\omega_i\ ;$$

producing:

(5) $$\xi_k = \xi_{np} + \delta\xi\ ; \quad \eta_k = \eta_{np} + \delta\eta\ ; \quad \omega_k = \omega_{np} + \delta\omega\ .$$

We have the spherical coordinates and components of the velocity $\boldsymbol{S} = (\xi,\eta,\omega,\xi',\eta',\omega')_k^T$, the inertial spatial coordinates $\boldsymbol{r_k} = (X,Y,Z)_k^T$, according to eqs.(1) and components of the velocity $(\dot{X},\dot{Y},\dot{Z})_k$ [4, (4.38)], of the satellite for the moment t_k from eqs.(1), and the measured GPS coordinates of the GCP $\boldsymbol{R}_i = (X,Y,Z)_i$ .Then

(6) $$U_{ik}^0 = \begin{bmatrix}\gamma\\ \delta\\ \rho\end{bmatrix}_{ik} = \begin{bmatrix} arctg\ \dfrac{Y_k - Y_i}{X_k - X_i} \\ \arcsin\dfrac{Z_k - Z_i}{\sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2}} \\ \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} \end{bmatrix} .$$

By linearization of expressions (6) and taking into account the equation of floating differences, which is typical in using an orbital method, [4] we have:

(7) $$U_{ik} + dU_{ik} = U_{ik}^0(X_k^0,Y_k^0,Z_k^0,X_i^0,Y_i^0,Z_i^0) + \frac{\partial U_{ik}^0}{\partial(X^0,Y^0,Z^0)_k}$$
$$\frac{\partial(X_k^0,Y_k^0,Z_k^0)}{\partial(\xi,\eta,\omega)_k^0} \cdot \frac{\partial(\xi,\eta,\omega)_k^0}{\partial(\xi_0,\eta_0,\omega_0,\xi_0',\eta_0',\omega_0')_k} \,.\, d(\xi_0,\eta_0,\omega_0,\xi_0',\eta_0',\omega_0')_k +$$
$$+ \frac{\partial U_{ik}}{\partial(X_i,Y_i,Z_i)} \,.\, d(X_i,Y_i,Z_i)\ .$$

The equation of the floating differences after abbreviation by cancellation in expressions (7), is written in the form:

(8) $$\mathbf{V}_{U_{ik}} = (\boldsymbol{A}_{ik}\ \boldsymbol{I}_k\ \boldsymbol{J}_k\ \boldsymbol{B}_{ik})_j \begin{pmatrix} d\mathbf{S}_{0j} \\ d\mathbf{R}_i \end{pmatrix} + \boldsymbol{L}_{U_{ik}} = (\boldsymbol{G},\boldsymbol{B})_{kj} \begin{pmatrix} d\mathbf{S}_{0j} \\ d\mathbf{R}_i \end{pmatrix} + \boldsymbol{L}_{U_{ik}}; \quad \boldsymbol{P}_{U_{ik}}\ ,$$

where:

(9) $$\boldsymbol{G}_{kj} = \boldsymbol{A}_{ik}\ \mathbf{I}_{kj}\ \mathbf{J}_{kj}\ ,$$

(10) $$\boldsymbol{L}_{U_{ik}} = U'_{ik} - U^{0}_{kj}\ (\mathbf{R}_i \mathbf{S}_{oj}, t_k - t_0)\ ,$$

U'_{ik} - is determined from the coordinates of GCP - i ; and satellite - k ,

U_{kj} - are extrapolated coordinates of satellite -k ;

(11) $$\boldsymbol{A}_{ik} = -\boldsymbol{B}_{ik} = \frac{\partial U^0_{ik}}{\partial(X^0,Y^0,Z^0)} = \begin{bmatrix} \dfrac{-\cos\gamma\sin\delta}{\rho} & \dfrac{\cos\gamma\sin\delta}{\rho} & \dfrac{\cos\delta}{\rho} \\ \dfrac{\sin\gamma}{\rho} & \dfrac{-\cos\gamma}{\rho} & 0 \\ \cos\gamma\cos\delta & \sin\gamma\cos\delta & \sin\delta \end{bmatrix}^0_{ik} = \begin{bmatrix} \dfrac{-\Delta x\Delta z}{s\rho^2} & \dfrac{\Delta y\Delta z}{s\rho^2} & \dfrac{s}{\rho^2} \\ \dfrac{\Delta y}{\rho^2} & \dfrac{-\Delta x}{\rho^2} & 0 \\ \dfrac{\Delta x}{\rho} & \dfrac{\Delta y}{\rho} & \dfrac{\Delta z}{\rho} \end{bmatrix}^0_{ik}$$

where:

(12) $$\begin{cases} \Delta x_{ik} = X^0_k - X_i \quad \Delta y_{ik} = Y^0_k - Y_i \quad \Delta z_{ik} = Z^0_k - Z_i\ , \\ \rho_{ik} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}\ , \\ s_{ik} = \sqrt{\Delta x^2 + \Delta y^2}\ ; \end{cases}$$

(13) $$\mathbf{I} = \frac{\partial(X^0_k, Y^0_k, Z^0_k)}{\partial(\xi,\eta,\omega)^0_k} = \begin{vmatrix} \dfrac{S_0}{F_0}\xi\cos\omega & -\dfrac{k_0}{F_0}\eta\cos\omega & -F_0\sin\omega \\ \dfrac{S_0}{F_0}\xi\sin\omega & -\dfrac{k_0}{F_0}\eta\sin\omega & -S_0\sin\omega \\ \eta & \xi & 0 \end{vmatrix}_k ,$$

for S_0, k_0, F we have the expressions:

(14) $$S_0 = 1 - \eta_0^2\ ; \qquad k_0 = c^2 + \xi_0^2\ ; \qquad F_0 = \sqrt{S_0}k_0\ ;$$

(15) $$\mathbf{J} = \frac{\partial(\xi,\eta,\omega)^0_k}{\partial(\xi_0,\eta_0,\omega_0,\xi_0',\eta_0',\omega_0')_k} = \begin{bmatrix} \dfrac{\partial\xi_k}{\partial\xi_0} & \dfrac{\partial\xi_k}{\partial\eta_0} & \cdots\cdots\cdots & \dfrac{\partial\xi_k}{\partial\eta'_0} & \dfrac{\partial\xi_k}{\partial\omega'_0} \\ \dfrac{\partial\eta_k}{\partial\xi_0} & \dfrac{\partial\eta_k}{\partial\eta_0} & \cdots\cdots\cdots & \dfrac{\partial\eta_k}{\partial\eta'_0} & \dfrac{\partial\eta_k}{\partial\omega'_0} \\ \dfrac{\partial\omega_k}{\partial\xi_0} & \dfrac{\partial\omega_k}{\partial\eta_0} & \cdots\cdots\cdots & \dfrac{\partial\omega_k}{\partial\eta'_0} & \dfrac{\partial\omega_k}{\partial\omega'_0} \end{bmatrix} .$$

The values of the coefficients of matrix (12) are defined from [4, (4.18)].

The vectors $d\mathbf{S}_{0k}$ and $d\mathbf{R}_i$ have the form:

(16) $$d\mathbf{S}_{0k} = (d\xi_0, d\eta_0, d\omega_0, d\xi'_0, d\eta'_0, d\omega_0)^T\ ,$$

(17) $$d\mathbf{R}_i = (dX_i, dY_i, dZ_i)^T\ ,$$

(18) $$\boldsymbol{S}_k^1 = \boldsymbol{S}_{0k} + d\boldsymbol{S}_k^1 \ ,$$

(19) $$\boldsymbol{R}_i^1 = \boldsymbol{R}_i + d\boldsymbol{R}_i^1 \ .$$

In this case, when the GCP are defined by GPS measures , $d\boldsymbol{R} = 0$ could be placed in (19) , after which from eqs.(1) it is obtained :

(20) $$\boldsymbol{V}_{U_{kj}} = \boldsymbol{G}_{kj} d\boldsymbol{S}_{kj}^1 + \boldsymbol{L}_{U_{kj}} \ ; \qquad \boldsymbol{P}_{U_{ik}} \ .$$

The solution of the equation of floating differences (20), provides to obtain the unknown corrections $d\boldsymbol{s}$ from (16) and from (1) the values $\boldsymbol{r}_k = (X^1, Y^1, Z^1)_k$ in inertial coordinate system which could be taken as a first approximation, namely:

(21) $$\begin{cases} \xi_k^1 = \xi^0 + d\xi \ , \\ \eta_k^1 = \eta^0 + d\eta \ , \\ \omega_k^1 = \omega^0 + d\omega \ ; \end{cases}$$

(22) and from (1) it is obtained: $\boldsymbol{r}_k = (X^1, Y^1, Z^1)_k$.

3. Fluxional specification of the coordinates of the satellite for the moments t_k and defining the Oiler's elements (Ω, i, ω,).

Upon having obtained in first approximation the coordinates of the satellite $(X^1, Y^1, Z^1)_k$, in the moments of taking the image t_k (k = 1,2…n) and having the defined coordinates (X_i, Y_i, Z_i) from the GCP, it is possible to accomplish the ultimate goal - time-coordinate georeference of the space image with the defining of the decode places and identified points (fig.1) in the Geenwich geocentric coordinate system OXYZ, firmly related with the rotating Earth. We will accept: that the beginning of the coordinate system O to coincide with the Earth's mass center or with the Earth's /referent/ ellipsoid center; the movement of the poles is accounted for.

Below we shall use the fundamental equation of space photogrametry [2, 9, 12] (fig.2) for fluxional specification of the satellite orbit:

$$(23)\qquad \boldsymbol{\rho}_{ki} = -(\boldsymbol{r}_k - \boldsymbol{R}_i) \qquad k=1,2.\ldots n \qquad i=1,2.\ldots\ldots 10$$

In each direction of the centric-satellite distance-vector $\boldsymbol{\rho}^0_{ki}$ to the GCP i, 'intersects' the topographic image in $\bar{i}$, which provides for the fundamental equation (22) to be presented in the form of the Greenwich coordinate system, by the expression (fig.2 and 3):

$$(24)\qquad \begin{vmatrix} \bar{x}_{\bar{i}} \\ \bar{y}_{\bar{i}} \\ -f_t \end{vmatrix} = \frac{\Delta_{ki}}{\rho_{ki}} \boldsymbol{S_k P}_k \begin{vmatrix} X_i - X_k \\ Y_i - Y_k \\ Z_i - Z_k \end{vmatrix},$$

where:

$\bar{x}_{\bar{i}}$ $\bar{y}_{\bar{i}}$ f_t - are the coordinates in the topographic coordinate system of the space image (fig.3) ;

$\frac{1}{m} = \frac{\Delta_{ki}}{\rho_{ki}}$ - a scale coefficient ;

$\Delta_{ki} = \sqrt{x_k{}^2 + y_k{}^2 + f_t^2}$ - a scale factor ;

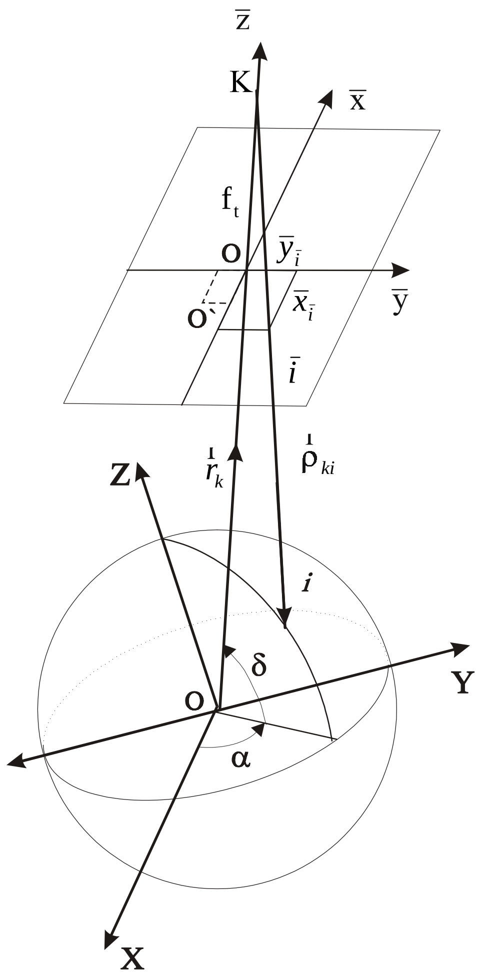

Fig. 3

(25) $$S_k = \begin{vmatrix} \cos S & -\sin S & 0 \\ -\sin S & \cos S & 0 \\ 0 & 0 & 1 \end{vmatrix}$$ - is a matrix for transformation in to Greenwich coordinate system.

For the orthogonal matrix $\boldsymbol{P}_k$ we have :

(26) $$\boldsymbol{P}_k = (\boldsymbol{P}_0 \ \boldsymbol{P}_i^*)^T$$

where $\boldsymbol{P}_0$ is an orthogonal matrix giving the orientation between the topographic and the star coordinate system ;

$\boldsymbol{P}_i^*$- operator giving the orientation of the star image towards the inertial.

$\boldsymbol{P}_k$ in matrix form is:

(27) $$\boldsymbol{P}_k = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

where a_i, b_i, c_i $(i = 1, 2, 3)$ are the guiding cosines from the Oiler's angles, which in [2, (14)] are given by the indexes: Ω', i', ω' .

Excluding the scale coefficient $\frac{1}{m}$, equation (24) could be written in colinear form :

(28) $$\begin{cases} x_{ki} = -f_t \dfrac{a_1(X_i - X_k) + a_2(Y_i - Y_k) + a_3(Z_i - Z_k)}{c_1(X_i - X_k) + c_2(Y_i - Y_k) + c_3(Z_i - Z_k)} \\ y_{ki} = -f_t \dfrac{b_1(X_i - X_k) + b_2(Y_i - Y_k) + b_3(Z_i - Z_k)}{c_1(X_i - X_k) + c_2(Y_i - Y_k) + c_3(Z_i - Z_k)} \end{cases} .$$

The linearisation of the equation (24), respectively (28), for every pictured GCP *i*, on the topographical image $\bar{i}$ with coordinates $\bar{x}_{k\bar{i}}$ and $\bar{y}_{k\bar{i}}$ yields:

(29) $$\bar{u}_{ki} + d\bar{u}_{ki} = \bar{u}_{ki}^0(\bar{x}_{ki}^0, \bar{y}_{ki}^0) + \frac{\partial \bar{u}_{ki}}{\partial(\Omega, i, \omega)_k} d(\Omega, i, \omega)_k + \\ + \frac{\partial \bar{u}_{ki}}{\partial(X, Y, Z)_k} d(X, Y, Z)_k + \frac{\partial \bar{u}_{ki}}{\partial(X, Y, Z)_i} d(X, Y, Z)_i$$

where:

$\bar{u}_{ki} = (\bar{x}_{ki}, \bar{y}_{ki})^T$ - the calculated values, with the obtained after each itegration coordinates according to (28);

$\bar{u}_{ki}^0(x_{ki}^0, y_{ki}^0)$ - the measured coordinate from the of GCP 's image.

Because of some considerations shared above for the coordinates of GCP, we shall assume the equation of floating differences from (29):

(31) $$\boldsymbol{V}_{\bar{u}_{ki}} = (\boldsymbol{G}_k, \boldsymbol{B}_k)\begin{pmatrix} d\boldsymbol{s}_k \\ d\boldsymbol{r}_k \end{pmatrix} + \boldsymbol{L}_{\bar{u}_{ki}}; \qquad \boldsymbol{P}_{\bar{u}_{ki}} \;;$$

(32) $$d\boldsymbol{s}_k = (d\Omega, di, d\omega)_k^{\;T} \;;$$

$$d\boldsymbol{r}_k = (dX, dY, dZ)_k^{\;T} \;;$$

(33) $$\boldsymbol{L}_{\bar{u}_{ik}} = \bar{\boldsymbol{u}}_{ki}^0 - \bar{\boldsymbol{u}}_{ki} \;;$$

(34) $$\boldsymbol{G}_k = \frac{\partial \overline{\boldsymbol{u}}_{ki}}{\partial(\Omega, i, \omega)_k} \quad ; \quad \frac{\partial \overline{\boldsymbol{u}}_{ki}}{\partial(X, Y, Z)_k} \; .$$

By equations (31) we make several consecutive iterations, until $d\boldsymbol{s}_k$ and $d\boldsymbol{r}_k$ become smaller than the initial quantity ε_0, i.e. we have:

(35) $$\begin{cases} \Omega_k = \Omega_k^1 + d\Omega_k^2 + d\Omega_k^3 + \ldots , & X_k = X_k^1 + dX_k^2 + dX_k^3 + \ldots \\ i_k = i_k^1 + di_k^2 + di_k^3 + \ldots , & Y_k = Y_k^1 + dY_k^2 + dY_k^3 + \ldots \\ \omega_k = \omega_k^1 + d\omega_k^2 + d\omega_k^3 + \ldots , & Z_k = Z_k^1 + dZ_k^2 + dZ_k^3 + \ldots \, . \end{cases}$$

where $n = 2, 3, \ldots\ldots$,

$$d\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{n}} = (d\Omega_k^n, di_k^n, d\omega_k^n)^T < \varepsilon_0^1 \quad , \qquad d\boldsymbol{r}_{\boldsymbol{k}}^{\boldsymbol{n}} = (dX_k^n, dY_k^n, dZ_k^n)^T < \varepsilon_0^2 \; .$$

4. Coordinate georeference of the control points from the space image in the Greenwich coordinate system.

Upon obtaining the specified values for the external-orientation of the space images according to (35), it is possible to define the coordinates of the already known control points from the topografic image in the Greenwich coordinate system, by which we could remove the geometric deformations. According to the studies in [1,12,13,15], it is proved that the maximum control points must be 35-40, moreover, their optimal position on the photo is shown.

In (29) we could put $d\boldsymbol{s}_k = (d\Omega, di, d\omega)_k^{\ T} = 0$, $d\boldsymbol{r}_k = (dX, dY, dZ)_k^{\ T} = 0$, and we will obtain the following equations of floating differences:

(36) $$\boldsymbol{V}_{\overline{u}_{ki}} = \boldsymbol{A}_i d\boldsymbol{R}_i + \boldsymbol{L}_{\overline{u}_{ki}} \quad ;$$

(37) $$\begin{cases} \boldsymbol{A}_i = \dfrac{\partial u_{ki}}{\partial(X, Y, Z)_i} = \dfrac{\partial(\overline{x}_{ki}, \overline{y}_{ki})}{\partial(X, Y, Z)_i} \\ d\boldsymbol{R}_i = (dX, dY, dZ)_i^T \quad ; \end{cases}$$

(38) $$\boldsymbol{L}_{\overline{u}_{ki}} = \overline{\boldsymbol{u}}_{ki}^0 - \overline{\boldsymbol{u}}_{ki} = \begin{vmatrix} \overline{x}_{ki}^0 - \overline{x}_{ki} \\ \overline{y}_{ki}^0 - \overline{y}_{ki} \end{vmatrix} \quad ,$$

$\bar{x}_{ki}^{0}$, $\bar{y}_{ki}^{0}$ -are the measured values of the control points from the space image ;

$\bar{x}_{ki}$, $\bar{y}_{ki}$ -the calculated values of the control points according to (28).

It necessary, the equation of the floating differences could be solved by iteration, by introducing in (28) the coordinate of the control points obtained after the last iteration.

Soures: